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ABSTRACT

GRADES OR AGES: Grade 12. SUBJECT MATTER: Mathematics. ORGANIZATION AND PHYSICAL APPEARANCE: The subject matter is presented in four columns: major areas, significant outcomes, observations and suggestions, and films and references. The topics include: sets-relations-functions, circular functions, graphs of circular functions, inverses of circular functions, trigonometric functions or angle measures, introduction to vectors, the polar plane, complex numbers, and infinite series. The guide is mimeographed and spiral bound, with a soft cover. OBJECTIVES AND ACTIVITIES: Objectives for each major area are stated in behavioral terms. Activities are suggested but not listed in detail. INSTRUCTIONAL MATERIALS: Textbook references are given for each major area and there is a brief bibliography. No audio-visual materials are listed. STUDENT ASSESSMENT: Tests on major areas, with answers are included. (MBM)

MATHEMATICS CURRICULUM GUIDE

MATHEMATICS IV

Trigonometry - Vectors

Complex Numbers - Infinite Series

SCHOOL CITY OF GARY
Gary, Indiana
1969

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Procedure

The Department Representative Committee, working in their sessions, review the mathematics program and recommend specific improvements needed to strengthen instruction. Some of the recommendations require the service of special committees.

This supplement was proposed to up-date the Mathematics IV curriculum by the Department Representative Committee. The writing of the first draft took place during the summer of 1968. The draft was submitted to the mathematics faculty for suggestions for incorporation in this edition. All materials were reviewed and edited by the mathematics consultant.

FOREWORD

This mathematics guide represents another step in the on-going process of developing relevant mathematics curriculum in the Gary Public Schools. The guide outlines major areas of study for senior students enrolled in college preparatory mathematics. It defines significant outcomes in behavioral terms, provides teaching suggestions, identifies learning materials, and includes evaluation items which can aid in determining students' achievement. It is the result of cooperative efforts by the teaching and supervisory staffs. It is written in the spirit of what might be considered the overall mathematics teaching goals: 1) To learn to read mathematics to acquire fundamental concepts 2) To do mathematics by the development of skill in handling symbols and computations 3) To develop ability to solve problems through observation, selection, generalization, and conceptualization. Underlying this, as in the preparation of other curriculum guides, is the assumption that teachers will be governed in their course objectives and instructional materials by concepts pupils have acquired.

Appreciation is expressed to each individual, to the committee, and supervisory personnel who have contributed to the development and strengthening of the mathematics curriculum. We hope that the mathematics teachers will use this guide diligently, evaluate the results carefully, and share in identifying further revisions that will keep the program moving forward in accord with the many worthwhile changes occuring in the field of mathematics education.

GORDON McANDREW
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P R E F A C E

This guide is the outgrowth of considerations determined to be vital to the completion of secondary mathematics education. The guide reflects, in general, the considerations of recent textbook writers and curriculum committees concerned with mathematics education for the senior year of high school. As any worthwhile guide should, it reflects the experienced opinions of teachers who were directly involved in the teaching of this course in Gary.

A reader using the guide will notice that each unit is outlined in some detail. Significant outcomes have been stated in behavioral terms. Suggestions for presentations and references of current textbooks that can be helpful as other alternatives to mastering concepts have been included. Tests to accompany the various units have been included as samples of items considered to test whether or not some of the behaviors specified in the units have been achieved. Achievement of students must also be considered in light of oral responses to questions and written solutions to problems in classroom settings.

Any guide or textbook in mathematics can contribute to the learning of students to the extent that the teacher adequately prepares lessons and presents them in a clear and interesting manner. Appropriate use of chalkboards, graph paper, special work sheets, overhead projectors and either commercially or teacher-prepared projectuals can help to clarify and expand ideas. Not least of importance to successful learning of this subject is adequate application to the study by the students through faithfully performed or attempted and appropriately assigned homework. Finally an atmosphere must prevail in the classroom that can encourage students' questions and sufficient discussions to clarify and build on ideas presented.

It is the wish of the committee preparing this guide that it be helpful to teachers in organizing their course work and that it leads to satisfaction for the teachers teaching the course and students learning the course through success.

TABLE OF CONTENTS

MAJOR AREAS	SIGNIFICANT OUTCOMES
I. Sets - Relations - Functions A. Sets and Set Notation B. The Real Numbers C. Open Sentences D. Graphing in the Plane E. Sets of Ordered Pairs F. Composition and Inverse of Functions	1. Specify a set by roster or rule using set builder notations. 2. Determine union and intersection of given sets. 3. Identify elements of specified subsets of the set of real numbers; i.e., natural numbers, rational numbers, etc. 4. Write true sentences involving the relations $=$, $>$, $<$, between given real numbers and real number variables. 5. Given specified sets, draw their graphs on a number line. 6. Specify real number properties which justify mathematical sentences. 7. Find solution sets for first and second degree open sentences by using real number properties and by graphing. 8. State domain and range of a given relation. State whether or not the relation is a function. 9. Given two functions, write values for their sums, differences, products, and quotients. 10. Write the composite of two or three given functions, and state the domain and range of the result. 11. Given a relation, write its inverse and state the domain and range of the inverse relation. 12. Given a function, write its inverse and state whether or not the relation formed is a function.

OBSERVATIONS AND SUGGESTIONS	FILMS AND REFERENCES
I. Sets - Relations - Functions	13 Chapter 1 1 Chapter 1 8 Chapter 1 10 Chapter 1 11 Chapter 1 12 Chapter 1

I. Sets - Relations - Functions

Any student who leaves this major area without a mastery of each of the specified outcomes will experience serious difficulty in later areas of this course. Even though students have studied much of the information included in this major area, the presentation here is aimed at useful concepts for the study of circular functions.

Set builder notation should be emphasized because it is used extensively to develop concepts throughout this course. Venn diagrams, while of use in studying set relations, are considered of lesser importance. When students are asked to graph sets, include examples which require the use of unions and intersections of sets and also those which include absolute value.

Emphasize the limitations of the domain of relations and functions. When introducing the inverse of a relation one might wish to discuss the identity relation first. (See SMSG text Elementary Functions.)

It is recommended that the study of degree and radian measure be deferred until just prior to the introduction of the trigonometric functions. Sine and cosine functions are first defined in terms of real numbers. Radian and degree measures seem to interfere with this development.

For students of sufficient maturity, construction of mathematical induction arguments is suggested.

For students of considerably less maturity a review of preliminary concepts listed in reference (8) may be advisable.

MAJOR AREAS	SIGNIFICANT OUTCOMES

MAJOR AREAS

II. Circular Functions

A. Periodic Functions

B. Basic Circular Functions

1. Sine and cosine functions
2. Values of cos x and sin x for special values of x

C. Basic Circular Function Properties

1. Addition properties of cosine
2. Identities
3. Addition properties of sine
4. Other properties of sine and cosine
5. Use of the tables of values of sin x and cos x

D. Additional Circular Functions

1. Tangent
2. Secant
3. Cosecant
4. Cotangent

E. Identities

SIGNIFICANT OUTCOMES

1. Given a periodic function, state the fundamental period.
2. Given a table of values of x and f(x), where f is periodic, extend the table by using the relation $f(x) = f(x + ka)$.
3. Given an arc of a unit circle with length x and endpoints at (1, 0) and at (u,v), state the functional relationship between x and u and between x and v.
4. State the domain and range of the six basic circular functions.
5. Use the relationships $\cos(x + 2k\pi) = \cos x$ and $\sin(x + 2k\pi) = \sin x$ to write expressions for cos x and sin x where $0 \leq x \leq 2\pi$ and k is an integer.
6. Given $\cos^2 x + \sin^2 x = 1$, a specified quadrant, and value of either cos x or sin x write the value of the function whose value is not given.
7. Without refering to a table, state values of sin x and cos x for $x = 0, \frac{\pi}{6}, \frac{\pi}{4}$ and multiples thereof, but where $0 \leq x \leq 2\pi$.
8. Construct a table with values of sin x and cos x for $x = 0, \frac{\pi}{6}, \frac{\pi}{4}$ and multiples thereof, but where $0 \leq x \leq 2\pi$.
9. Use addition properties of cos and sin and identities derived therefrom to write values of these functions for specified numbers.
10. Write identities from addition properties of cos and sin.
11. Verify that given sentences are identities.
12. State the definitions of the functions tan, sec, csc, and cot, their fundamental period, and whether each is an even or odd function.

OBSERVATIONS AND SUGGESTIONS	FILMS AND REFERENCES
II. Circular Functions	13 Chapter 2 5 Chapter 6 8 Chapter 2, 3 10 Chapter 5 11 Chapter 3

II. Circular Functions

Graphing periodic function clarifies the repetitive character of these functions.

If a student understands domain and range for circular functions as they are introduced, he will be equipped to readily understand some of the derivations later developed in this major area. For example, if the domain is the set of reals, the domain and range are unchanged in the substitution of $(-x)$ for (x), hence the function is unchanged.

When deriving the basic identities, explain that the substitution of $\frac{x}{2}$ for x or $x_2 + x_1$ for $x_2 - x_1$ can be accomplished because the function is left unchanged. The elements of the domain have not been changed nor have the corresponding elements of the range. However, not every substitution can be made (i.e. depending upon the domain, $\frac{1}{x}$ can not usually be substituted for x because the number 0 has been lost to the domain).

Tables should be constructed by the students in which they list values of the various functions for special reals. These should be the result of computations using the reduction formulas, basic identities and definitions. The purpose is to further understanding of the relationships between the functions, to understand the strength and significance of definitions and the use of the basic identities. The idea of symmetry on the unit circle helps students understand these computations and derivations of identities.

It is important to have students write some of the more complex derivations in order to understand the parts rather than to memorize the whole process.
(e.g. $\tan(x_2 + x_1)$, $\cos(x_2 \pm x_1)$ etc.)

OBSERVATIONS AND SUGGESTIONS (CONT.)

When proving identities, the student must learn to use the many skills, definitions, etc. that are available, learn to begin again when frustrated, and explore all avenues toward the solution.

MAJOR AREAS	SIGNIFICANT OUTCOMES

III. Graphs of Circular Functions

- A. Graphs of Cosine and Sine Functions
- B. Graphs of Other Circular Functions
- C. Graphs of Functions Involving Sine and Cosine
- D. Graphs of Sums of Functions by Addition of Ordinates
- *E. Applications
  1. Uniform circular motion
  2. Simple harmonic motion

1. In a Cartesian plane draw the graph of
   a. $\{(x,y): y = \cos x\}$
   b. $\{(x,y): y = \sin x\}$
   c. $\{(x,y): y = \tan x\}$
   d. $\{(x,y): y = \cot x\}$
   e. $\{(x,y): y = \sec x\}$
   f. $\{(x,y): y = \csc x\}$
2. List special characteristics of each graph, e.g. maximum points, minimum points, points at which the graph intersects the axis, points of discontinuity, etc.
3. Given the sets
   $\{(x,y): y = a \cos (bx + c) + d\}$
   $\{(x,y): y = a \sin (bx + c) + d\}$
   a. Draw the graph of each in the Cartesian plane.
   b. Describe the change in the graph of each resulting from a change in any of the variables a, b, c, or d.
   c. State the amplitude, period, the phase shift (amount and direction) for any combination of a, b, c, or d.
   d. Describe the effect on each graph when $a < o$.
4. Use the graphs of sin x and cos x to illustrate relationships such as $\cos (\pi + x) = -\cos x$, $\sin (-a) = -\sin a$, etc.

*Optional

OBSERVATIONS AND SUGGESTIONS	FILMS AND REFERENCES
III. Graphs of Circular Functions	13 Chapter 33 5 Chapter 6 8 Chapters 3, 5, 6 12 Chapter 9

Students should progress from plotting points to sketching curves. They might first plot points in the interval $0 \leq x \leq 2\pi$. They should note the maximum point, the minimum point, and the point at which the graph intersects the axis. Using these points and their knowledge of the periodic character of the functions, they should extend the graph over several intervals. This development should progress so that the student should be aware of the special characteristics (amplitude, period, etc.) which will enable him to sketch the curve quickly.

Added insight may be achieved if the student describes changes in the domain and range resulting from changes in a, b, c, and d in graphs of $\{(x,y): y = a \cos (bx + c) + d\}$.

Several graphs may be drawn on one pair of axes. If each graph is sketched in a different color, students get a clearer idea of what affects amplitude, period, and phase shift than if each graph were drawn separately.

A compass is very helpful when plotting coordinates geometrically, e.g., $f(x) = 2 \sin x$, $f(x) = \sin x + \cos x$, etc.

Perceptive and inquisitive students will find the topics on uniform circular, motion simple, harmonic motion and pure waves very interesting and stimulating. These are receiving heavier emphasis in science.

THE USE OF AN OVERHEAD PROJECTOR, APPROPRIATE TRANSPARENCIES AND OVERLAYS IS PRACTICALLY INDISPENSABLE FOR TEACHING THIS MAJOR AREA.

MAJOR AREAS	SIGNIFICANT OUTCOMES
IV. Inverses of Circular Functions A. Inverses of Sine and Cosine B. Inverses of Other Circular Functions C. Solution Sets of Open Sentences Involving Sin x and Cos x D. Solution Sets of Open Sentences Involving Other Circular Functions	1. Given a circular function define its inverse i.e. $\cos^{-1}x = \{(x,y) : x = \cos y\}$ 2. State the domain and range of the inverse of each circular function. 3. For specified values of x, write the sets: $\{y : y = \cos^{-1}x\}$ $\{y : y = \sin^{-1}x\}$, etc. 4. For specified values of x, write values for the functions $\mathrm{Cos}^{-1}x$ and $\mathrm{Sin}^{-1}x$, etc. 5. Find values of expressions such as Arctan (-1), $\mathrm{Tan}^{-1}(\cot \frac{\pi}{4})$ 6. Use identities and properties of inverses of functions and relations to solve open sentences involving the circular functions.

OBSERVATIONS AND SUGGESTIONS	FILMS AND REFERENCES

IV. Inverses of Circular Functions

In considering the inverse of a circular function, clarity can be achieved by analyzing the domain and range. The distinction between the <u>inverse of a function</u> and the <u>inverse function</u> is based on this. In obtaining the <u>inverse of a function</u>, the entire domain and range are interchanged; the mapping is one-to-many; hence, this is a relation. In considering the <u>inverse function</u>, the range is restricted after the interchange has been made, to make the mapping one-to-one and hence, a function.

Graphs should be utilized to enhance the understanding of the abstract concepts in this major area.

The teacher may wish to develop the following as theorems:

$$\sin(\mathrm{Cos}^{-1}u) = \cos(\mathrm{Sin}^{-1}u) = \sqrt{1 - u^2}$$

$$\sin(\mathrm{Tan}^{-1}u) = \frac{u}{\sqrt{1 + u^2}}$$

$$\cos(\mathrm{Tan}^{-1}u) = \frac{1}{\sqrt{1 + u^2}}$$

It should be pointed out to the students that there are alternate forms of notation which can be used to express the inverses of the functions.

Caution students that:

$$\cos(\mathrm{Tan}^{-1}x + \mathrm{Sin}^{-1}x) \neq \cos(\mathrm{Tan}^{-1}x) + \cos(\mathrm{Sin}^{-1}x)$$

Films and References:

13 Chapter 4
5 Chapter 13
8 Chapter 8
11 Chapter 3
12 Chapter 10

MAJOR AREAS	SIGNIFICANT OUTCOMES
V. Trigonometric Functions of Angle Measures	1. Define radian measure in terms of the radius of a circle.
A. Angle Measure	2. Convert radian measure to degree measure and vice versa.
B. Trigonometric Functions and Identities	3. Describe the difference between the trigonometric functions and circular function.
C. Tables of Trigonometric Functions	4. Given angles whose measures are expressed in degrees or radians, write values of the trigonometric functions of the angles.
D. Reference Angles and Function Values	5. Use trigonometric tables and interpolation when necessary to find values of trigonometric functions and find measure of angles when the value of the function is given.
E. Equations and Inverse Functions	6. Use reference angles to find values of trigonometric functions of any angle.
F. Solution of Triangles	7. Given an angle θ in standard position with the terminal side determined either by coordinates of a point or by an equation, write values of cos θ, sin θ, tan θ.
1. Right	8. Given appropriate data, solve right triangles.
2. Law of cosines	9. Use the law of sines and the law of cosines to solve any triangle.
3. Area	
4. Law of sines	
5. The ambiguous case	

OBSERVATIONS AND SUGGESTIONS	FILMS AND REFERENCES
V. Trigonometric Functions of Angle Measures	13 Chapter 5 1 Chapters 6, 7 5 Chapters 1-5, 7, 8 8 Chapter 6 11 Chapters 4, 6 12 Chapters 2, 7, 8

A thorough discussion of range and domain of the trigonometric functions will help to avoid confusion with the circular functions. It may be of interest to the students that this is an example of how mathematics is applied to new situations. Sometimes a change of the domain can create a new system which is isomorphic to the first as with the circular functions and the trigonometric functions. The fact that the domain is now a set of angles allows for applications not possible before, e.g., reference angles and solutions of triangles. The use of radian measure of angles provides an excellent vehicle for the transition from circular functions to trigonometric functions.

When approximating values of trigonometric functions it is just as important to develop an understanding of the process of linear interpolation as it is to perform computation using this method.

Some studies define the trigonometric functions in terms of ratios between sides of a right triangle. In this study, however, the trigonometric ratios are results of the original definitions of the functions.

After this topic, a number of options are available to teachers. Among these are Units V through IX of this guide. Other options include a study of topics of analytical geometry such as the straight line, conic sections, translations and rotations, or a study of introductory probability and statistics. Approved outlines of such topics are being prepared for review by the Mathematics Department Chairmen for incorporation into this guide.

MAJOR AREAS	SIGNIFICANT OUTCOMES
VI. Introduction to Vectors A. Definition and Geometric Representation B. Basis Vectors C. Inner Product of Two Vectors *D. Free Vectors -- Navigation Applications *E. Applications to Forces	VI.1. Given a vector (x,y), draw its geometric representation. 2. Given a vector: a. State its norm b. Determine the measure of its direction angle c. State its negative d. Write the product of the vector and a given scalar 3. State the vector determined bv a given norm and direction angle. 4. Given two vectors, state: a. Their sum b. Their difference c. Their inner product (dot product) d. The angle between the vectors 5. Given a vector, state its scalar components. *6. Solve problems of navigation and/or force by applying vector concepts.
VII. The Polar Plane A. Definition and Description B. Polar Coordinates C. Graphical Representations	VII.1. Given a pair of polar coordinates, graph the corresponding point in the polar plane. 2. Given a pair of polar coordinates, write additional pairs which represent the same point. 3. Write equivalent polar coordinates for given Cartesian coordinates. 4. Express a given Cartesian equation as an equivalent polar equation. 5. Sketch some elementary curves in the polar plane.

*Optional

OBSERVATIONS AND SUGGESTIONS	FILMS AND REFERENCES
VI. Introduction to Vectors The study of vectors is included here because it is a unifying topic. It combines concepts learned in studying trigonometric functions with another view of the Cartesian plane. It serves as an additional preparation for a more meaningful study of complex numbers and the associated plane. The teacher should be aware of the great difference in notation and approach to the study of vectors from one author to another. Furthermore, special attention to the notation of a particular author should be exercised. Application of vectors might be more profitably studied in a course in physics or as a means of enrichment in independent study.	VI.13 Chapter 6 3 Chapter 4 6 Chapter 11 8 Chapter 10 9 Chapters 19-20
VII. The Polar Plane The study of polar coordinates develops an awareness in students that there are other possible coordinate systems for the plane. The study helps prepare them for a more meaningful study of the complex numbers and the associated plane. Graphing of polar equations is an integral part of this major area. (The use of polar coordinate graph paper is invaluable for doing this.) Students must develop the skill of sketching curves by observing critical points, by determing the characteristics of a curve in given intervals, and by noting symmetric relationships. They should use a minimum number of points in sketching. This skill is essential to successful study of advanced courses in mathematics. Some time can be profitably spent discussing domain and range of the relations studied in this topic.	VII.13 Chapter 6 2 Chapter 20 3 Chapter 12 4 Chapter 7 6 Chapter 5 7 Chapter 10, 2 8 Chapter 9

MAJOR AREAS	SIGNIFICANT OUTCOMES
VIII. Complex Numbers A. Definition and Operations B. Standard Form and Graphical Representation C. Polar Form *D. DeMoivre's Theorem *E. Roots of Complex Numbers	1. Given a complex number: a. Write its additive inverse b. Write its multiplicative inverse (if it exists) c. Write its modulus (or absolute value or norm) d. Write its amplitude e. Write its conjugate 2. Given two complex numbers: a. Find the sum b. Find the difference c. Find the product d. Find the quotient, if it exists 3. Graph complex numbers in the complex plane. 4. Solve equations having complex coefficients and roots. 5. Express i^n, where n is an integer, in terms of -1, 1 i, or -i. 6. Given complex numbers in standard form or as ordered pairs, write them in polar form.

*Optional

OBSERVATION AND SUGGESTIONS	FILMS AND REFERENCES
VIII. Complex Numbers The set of complex numbers is the set of all ordered pairs of real numbers and certain defined operations. The definition requires more than just to say that a complex number is an ordered pair. The system of complex numbers is not isomorphic to the system of vectors, and differences between the two should be noted such as multiplication and division. Further, while a point in the Cartesian plane corresponds to an ordered pair of real numbers, in the complex plane, a point corresponds to a single complex number. Teachers will also want to emphasize the lack of an order relation with complex numbers in contrast to the existence of this property with the real numbers. A meaningful study of DeMoivre's theorem presupposes an understanding of the principle of mathematical induction. It is not necessary to master this principal; however, to find roots of complex numbers using the theorem, reference(s) would be helpful in showing this.	13 Chapter 7 1 Chapter 9 2 Chapters 5-8 6 Chapter 5 8 Chapter 9 9 Chapter 20 11 Chapter 7

MAJOR AREAS	SIGNIFICANT OUTCOMES
IX. Infinite Series A. Sequences and Series B. Concept of Limit C. Geometric Sequences and Series D. Power Series E. Binomial Series *F. Series for sin x and cos x *G. Hyperbolic Functions	1. Write the first four or five terms of a sequence given the general term. Write the limit if it exists. 2. Given the first four terms of a sequence write a general term which would represent such a sequence. 3. Write the first four terms of a series which is given in sigma notation. 4. Given an expression of the form $\lim_{n \to \infty} S_n = b$ where S_n is some given sequence, find b if it exists. 5. Given a series write the first four terms of the sequence of partial sums for that series. 6. Find the value of: a. A finite geometric series expressed in sigma notation b. An infinite and convergent geometric series stated in sigma notation 7. Given some power series, state the interval of convergence. 8. Write the first four terms of a given binomial of the form: $(1 - x)^n$ where n is given. 9. Write a simplified form of a given expression involving factorial notation.

*Optional

OBSERVATIONS AND SUGGESTIONS	FILMS AND REFERENCES
IX. Infinite Series This topic is primarily included in this guide to provide students with introductory ideas of limits for possible use in future courses including the study of calculus. It will help to avoid confusion if the distinction between sequence and series is strongly emphasized whenever possible. Extreme care must be taken that the student develops <u>correct</u> understanding of the concepts of this section and that the symbols are used correctly. It is interesting to note that $n!$ can be defined by the two statements: $1! = 1$ and $n! = n(n - 1)!$ The better students may even enjoy proving $0! = 1$. Time may not permit the study of series for sin x and cos x for all members of a class. It might be of interest to students to assign a number of students to present this topic to the class since it is fascinating to see how values of sin x can be determined from a series.	13 Chapter 9 2 Chapter 2 3 Chapter 3 6 Chapter 10

MATHEMATICS IV

Suggested Test Questions for Major Area I

Circle to indicate whether the sentences in problems 1-5 are True or False.

All variables are real numbers.

1. T F $-5<|-5|$.

2. T F $|-n|<|n|$.

3. T F $|m|\geq|-m|$.

4. T F $x^2 + 3>3$.

5. T F $0\geq -n^2$.

Use number lines to graph each of the following:

6. $\{x: -1\leq x<2\}$

7. $\{x: |x - 2|\leq 1\}$

8. $\{x : x<3\}\cap\{x : x\geq -1\}$

Find solution sets over the real numbers for each of the following:

9. $y(5y - 3) = 18 + 5y(y + 3)$. ________________

10. $5(y - 2) = 6(y + 3)$. ________________

11. $2x^2 - 2x - 3 = 0$. ________________

12. Solve $3x + 4y^2 = 9$ for y in terms of x. ________________

For each of the following relations, state the domain, range and whether or not the relation is a function.

	DOMAIN	RANGE	FUNCTION (Circle one.)
13. $\{(x,y) : x^2 + 2y = 0\}$			yes no
14. $\{(x,y) : x = |2y|\}$			yes no
15. $\{(x,f(x)) : f(x) = 7 + x\}$			yes no

Given that $f(x) = x^2 - 3$ and $g(x) = 2x + 1$, find:

16. $f(3) + g(2) =$

17. $\dfrac{f(x + 1)}{g(x - 1)} =$

Given that $f = \{(x,y) : y = x^2 - x - 2\}$ and $g = \{(x,y) : y = x - 2\}$; express in terms of x:

18. $f - g =$

19. $f \cdot g =$

20. $f \circ g =$

MATHEMATICS IV

Suggested Test Questions for Major Area II

Given: $f = \{[x, f(x)] : x \text{ is an integer}\}$. Some values of x and f(x) are shown in the following table.

x	4	5	6	7	8	9	10	11	12
f(x)	0	1	2	3	0	1	2	3	0

Given that f is a periodic function with least fundamental period that is consistent with the given data, then (Exercises 1-4).

1. The fundamental period of f is ____________________.

2. $f(21) =$ ____________________.

3. If $f(x + ka) = f(x)$, where $k \in J$, what is the least value of a?

4. $f(4a) =$ ____________________.

5. Write values for each of the following in radical form:

   a. $\sin \frac{7\pi}{4}$ ____________________

   b. $\cos (\frac{+3\pi}{2})$ ____________________

   c. $\cos \frac{5\pi}{6}$ ____________________

6. Determine x if:

   a. $\cos x = 1, \frac{5\pi}{2} \leq x \leq 4\pi$ ______________

   b. $\cos x = \frac{\sqrt{3}}{2}, \pi \leq x \leq 2\pi$ ______________

   c. $\sin x = \frac{\sqrt{2}}{2}, \frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$ ______________

7. If $\cos x = \sin (-x)$, $0 \leq x \leq \pi$, then which one of the following is the value of x? (Circle your answer.)

   a. $\frac{\pi}{4}$ b. $\frac{3\pi}{4}$ c. $\frac{\pi}{2}$ d. $\frac{3\pi}{2}$

8. If $\cos x_2 = \frac{12}{13}$ and $\sin x_1 = \frac{8}{17}$, find a value for:

   a. $\sin (x_2 + x_1)$, $\frac{\pi}{2} < x_1 < \pi$, $0 < x_2 < \frac{\pi}{2}$ ____________________

   b. $\cos (\frac{\pi}{2} - x_1)$, $0 < x_1 < \frac{\pi}{2}$ ____________________

9. Which two of the following are identities? (Circle your answer.)

   a. $1-2 \cos^2 x = 2 \sin^2 x-1$

   b. $\cos (-x) \cos x = 1-\sin x \sin (-x)$

   c. $2 \cos x_1 \sin x_2 = \sin (x_1+x_2) - \sin (x_1-x_2)$

10. Determine the value of each of the following:

  a. tan x if cos x = $-\frac{4}{5}$ and sin $x > 0$ ____________________

  b. tan x if sin x = $\frac{12}{13}$, tan $x < 0$ ____________________

11. Which of the following are identities? (Circle your answer.)

  a. cot $(\pi - x)$ = cot x

  b. sec $(\frac{\pi}{2} + x) = -\csc x$

  c. csc $(\frac{3\pi}{2} - x) = -\sec x$

12. When simplified, $\sin^2 x(\cot^2 x + 1) =$

  a. sin x c. 1

  b. cos x d. 2

13. Given that sin x = $\frac{4}{5}$ and $\frac{\pi}{2} < x < \pi$, determine the value of sin 2x (Hint: sin 2x = 2 sin x cos x)

MATHEMATICS IV

Suggested Questions for Major Area III

1. Which one of the following statements is not true?

   a. $\sin(\pi - b) = -\sin(\pi + b)$

   b. $\sin\left(\frac{\pi}{2} + b\right) = -\sin\left(\frac{3\pi}{2} - b\right)$

   c. $\sin b = \sin(2\pi - b)$

   d. $|\sin(\pi - b)| = |\sin(b - \pi)|$

ALTERNATES:

a. $\{(x,y): y = 3\cos x + 1\}$

b. $\{(x,y): y = 2\sin x\}$

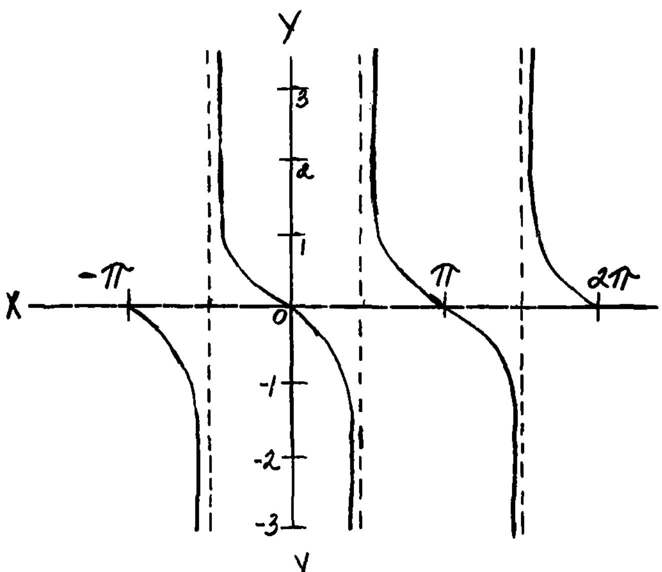
c. $\{(x,y): y = 2\cos x + 1\}$

d. $\{(x,y): y = \frac{1}{2}\cos x + 1\}$

e. $\{(x,y): y = \frac{1}{2}\sin x - 1\}$

f. $\{(x,y): y = -\cos x\}$

For each of the following graphs, choose the set represented, assuming $-\pi \leq x \leq 2\pi$

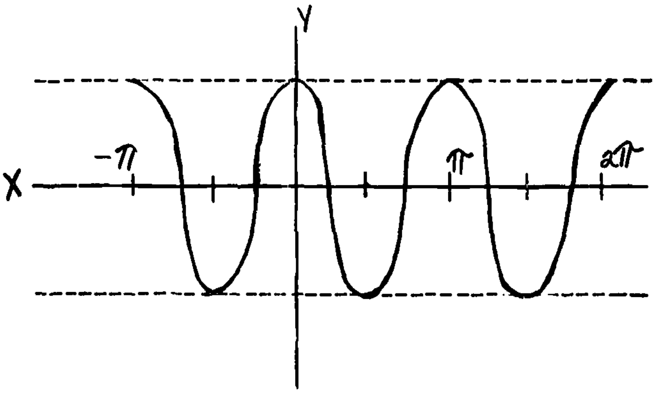
2. 

   a. $\{(x,y): y = \tan\left(x + \frac{\pi}{2}\right)\}$

   b. $\{(x,y): y = -\tan\left(x - \frac{\pi}{2}\right)\}$

   c. $\{(x,y): y = \cot\left(x + \frac{\pi}{2}\right)\}$

   d. $\{(x,y): y = \cot\left(x - \frac{\pi}{2}\right) + 1\}$

3. 

   a. $\{(x,y): y = \sin x + 1\}$

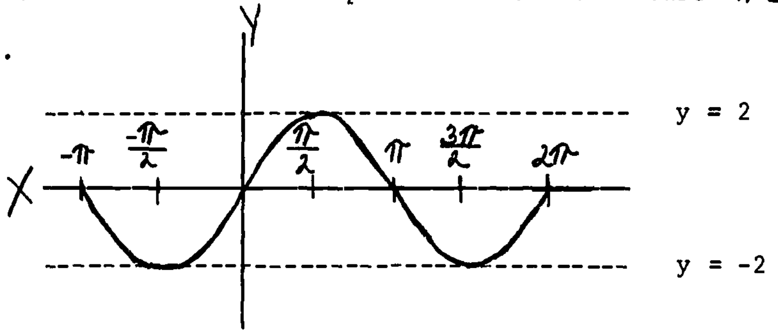
   b. $\{(x,y): y = 2\sin 2x\}$

   c. $\{(x,y): y = 2\cos 2x\}$

   d. $\{(x,y): y = \cos x + 1\}$

For each graph in problems 4-7 select from the given alternatives the function which it represents. (In each case $-\pi \leq x \leq 2\pi$)

4.



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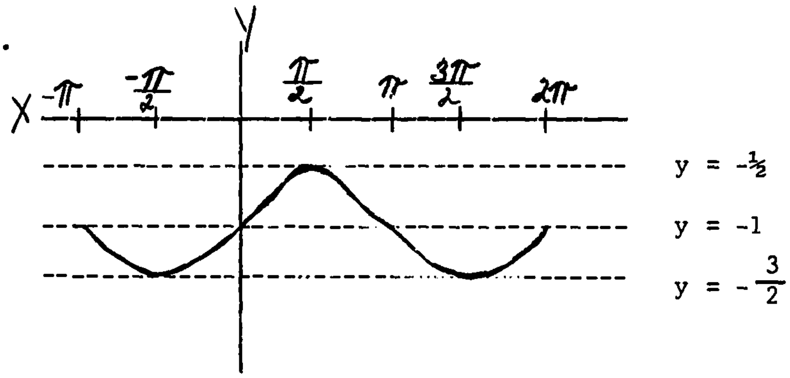

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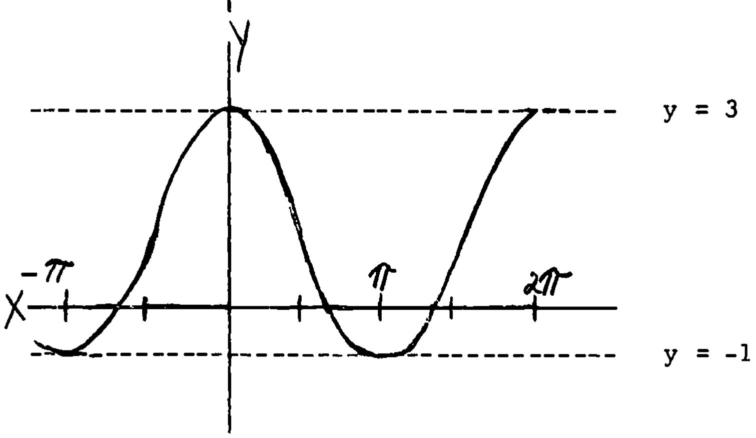

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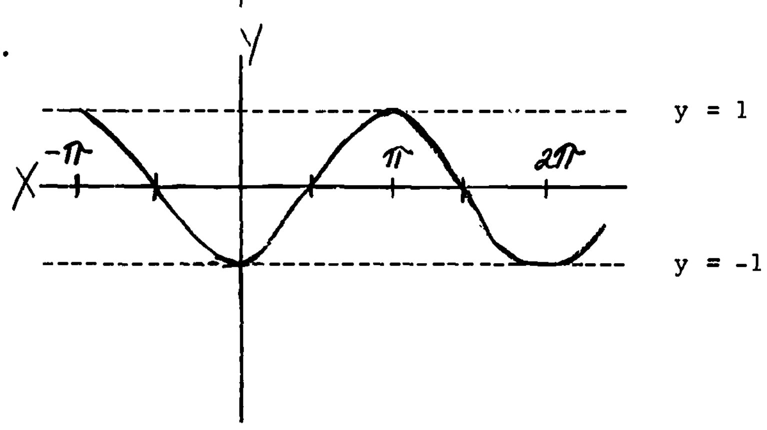


8. The graph of $\{(x,y) : y = 2 \cos (x-\frac{\pi}{3})\}$ has amplitude __________ and period ____________.

9. The graph of $\{(x,y) : y = -3 \sin (2x+\frac{\pi}{4}) +2\}$ has amplitude ____________ and period = __________.

10. For $-\pi \leq x \leq \pi$, what are the equations of the asymptotes of the graphs of:

a. $(x,y) : y = 3 \tan (2x + \frac{\pi}{4})$

b. $(x,y) : v = \sec (x - \frac{\pi}{3})$

MATHEMATICS IV

Suggested Test Questions for Major Area IV

List all values of x for each of the following in the interval $0 \leq x \leq 2\pi$:

1. $x = \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$
2. $x = \arctan(-\sqrt{3})$
3. $x = \sin^{-1}\frac{\sqrt{3}}{2}$
4. $x = \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$
5. $x = \cot^{-1}\left(\tan\frac{\pi}{3}\right)$

Determine the solution set over $\{x : 0 \leq x \leq 2\pi\}$ for each of the following:

6. $\sin 2x - 2\sin x = -$
7. $2\sin^2 x + \sin x - 1 = 0$
8. $2\cos x = 4\cos x \csc x$
9. $\tan^{-1} x + 2\tan^{-1} x = \frac{3\pi}{4}$

Circle the correct answer:

10. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) =$

 (A) $\frac{-\pi}{6}$ (B) $\frac{-\pi}{3}$ (C) $\frac{5\pi}{6}$ (D) $\frac{2\pi}{3}$

11. $\sin^{-1}\left(\cos\frac{5\pi}{6}\right) =$

 (A) $\frac{7\pi}{6}$ (B) $\frac{4\pi}{3}$ (C) $\frac{9\pi}{10}$ (D) $\frac{\pi}{6}$

12. $2\cos(\cos^{-1} x) =$

 (A) x (B) x^2 (C) $2x$ (D) 2

13. The solution set for $4\sqrt{2}\cos(x - 2\pi) + 4 = 0$ over $x : 0 \leq x \leq 2\pi$ is:

 (A) $\left\{\frac{3\pi}{2}, \frac{\pi}{2}\right\}$ (B) $\left\{\frac{3\pi}{4}, \frac{5\pi}{4}\right\}$ (C) $\left\{\frac{5\pi}{6}, \frac{11\pi}{6}\right\}$ (D) $\left\{\frac{4\pi}{3}, \frac{5\pi}{3}\right\}$

14. $\sin\left(\cos^{-1}\frac{3}{5}\right) =$

 (A) $\frac{2}{5}$ (B) $\frac{4}{5}$ (C) $\frac{-2}{5}$ (D) $\frac{3}{4}$

MATHEMATICS IV

Suggested Test Questions for Major Area V

1. If the radian measure of an angle is $\frac{11\pi^R}{3}$, its degree measure is ______________.

2. In a circle of radius 8, an arc of length $\frac{10\pi}{3}$ is intercepted by an angle whose degree measure is ______________.

3. Given that $\sin 32^\circ 20' = .5348$ and $\sin 32^\circ 30' = .5373$ and $\sin \theta = .5358$, find $m^\circ(\theta)$ to the nearest minute.

4. Given: $f(\theta) = 2 \text{ Cos } \theta$
   a. State the domain of f.
   b. State the range of f.

5. Write each of the following as a function of a positive acute angle:
   a. $\cos 300^\circ$
   b. $\sin (-225^\circ)$
   c. $\tan (-55^\circ)$

6. If the terminal side of the angle θ in standard position contains the point with coordinates (-3,4), determine the value of:
   a. $\sin \theta$
   b. $\cos \theta$
   c. $\tan \theta$

7. If θ is an angle in standard position whose terminal side is the graph of $(x,y) : x + 2y = 0,\ x \geq 0$, determine the value of:
   a. $\sin \theta$
   b. $\cos \theta$
   c. $\tan \theta$

For problems 8 and 9, we are given $\triangle ABC$ with $\underline{a}$ being the side opposite Angle A, $\underline{b}$, the side opposite Angle B and $\underline{c}$, the side opposite Angle C. (Write all irrational answers in simplest radical form) Use tables in textbook.

8. $b = 15$, $A = 30^\circ$, $C = 90^\circ$, find c.

9. $a = 20$, $c = 30$, $B = 40^\circ$, find b.

sin 400 = .6428
cos 40 = .7660

10. Two planes, one flying at 300 m.p.h. and the other at 400 m.p.h., left an airport at the same time. If the angle between their flight paths was 120°, then how far apart were they 2 hours later, assuming no change in course or speed?

MATHEMATICS IV

Suggested Test Questions for Major Area VI

1. Given the vector $V = (-\sqrt{3}, 1)$ and scalar $c = 2$:
   a. Determine the norm of V.
   b. Determine the direction angle θ of V.
   c. Draw the geometric vector corresponding to V.
   d. Draw the geometric vector corresponding to cV.

2. Determine the vector whose norm and direction angle are as follows:
   a. $\|v\| = 5, \ \theta = 240^\circ$
   b. $\|v\| = \sqrt{2}, \ \theta = 90^\circ$

3. Given: $V_1 = (3,1)$; $V_2 = (1, -3)$; $c_1 = 3$; $c_2 = -2$;

   Write the vector (ordered pair) corresponding to:

   a. $V_1 + V_2$
   b. $c_1 V_1 - c_2 V_2$
   c. $V_1 \cdot V_2$
   d. $c_2 V_1 \cdot c_1 V_2$

4. Determine the angle B, $0^\circ \leq B \leq 180^\circ$, between the vectors V_1 and V_2 if:
   a. $V_1 = \left(\frac{\sqrt{2}}{2} \ \frac{\sqrt{2}}{2}\right)$ and $V_2 = (1, 0)$
   b. $V_1 = (-2,5)$, and $V_2 = (10, 4)$

MATHEMATICS IV

Suggested Test Questions for Major Area VII

1. Which one of the following does <u>not</u> describe point A where A = $(5, 45^{o})$

   a. $(5, \frac{\pi}{4}^{R})$

   b. $\left(\frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2}\right)$

   c. $(-5, -45^{o})$

   d. $(-5, \frac{5\pi}{4}^{R})$

   e. $(5, 405^{o})$

2. Match the following equations and the correct description of the graph of that equation. Write the letter of the description next to the number of the equation:

   1. ___ $\rho = 5$
   2. ___ $\theta = \frac{3\pi}{2}^{R}$
   3. ___ $\rho = 0$
   4. ___ $\rho = 6 \sin \theta$
   5. ___ $\rho^2 = \cos 2\theta$
   6. ___ $\rho = 2\theta$
   7. ___ $\rho \cos \theta = 4$
   8. ___ $\rho = -9$
   9. ___ $\rho = 2 \sin 2\theta$

   a. A line perpendicular to polar axis at (4,0)

   b. Spiral of Archimedes

   c. Circle radius 3, center at $\left(3, \frac{\pi}{2}^{R}\right)$

   d. Four-leaved rose

   e. Circle with radius 1, center at $\left(-1, \frac{\pi}{2}^{R}\right)$

   f. Circle with center at pole and radius 5

   g. Line parallel to polar axis

   h. The pole

   i. A line through the pole and perpendicular to the polar axis

   j. Cardioid

   k. None of these

3. Write the polar equation $\rho = 5$ in Cartesian form.

4. Write the Cartesian equation $x + y - 4 = 0$ in polar form.

5. Write the Cartesian coordinates for the point whose polar coordinates are: $\left(5, \frac{5\pi}{6}^{R}\right)$

MATHEMATICS IV

Suggested Test Questions for Major Area VIII

1. Let $Z = (6, -2)$ Write:

   a. $-Z$

   b. Z^{-1}

   c. $\overline{Z}$

2. Let $Z_1 = (3, -1)$ $Z_2 = (-4, 5)$ Write:

   a. $Z_1 + Z_2$

   b. $Z_1 - Z_2$

   c. $Z_1 Z_2$

3. Let $Z_1 = 3 + 2i$, $Z_2 = 5 + 3i$. Write:

   a. $Z_1 - Z_2$

   b. $\dfrac{Z_1}{Z_2}$

4. Beside each power of <u>i</u> write the letter of its equivalent.

a. 1	____	i^{73}
b. -1	____	$(-i)^{13}$
c. i	____	$(-i)^{16}$
d. -i	____	$-i^{24}$
	____	i^{34}

5. Solve these open sentences

   a. $2-xi + 3x = 16y + yi - 12i$

   b. $(x + yi)(2 - 7i) = 34 - 13i$

   c. $4 \cos x + 4i \sin x = -2 + 2\sqrt{3}\, i$

6. Write in polar form

   a. $-2 + 2i$

   b. $-1 - \sqrt{3}\, i$

MATHEMATICS IV

Suggested Test Questions for Major Area IX

1. Write the first four terms of the sequence $\left\{(-1)^n \cos \frac{n\pi}{3}\right\}$

2. Write an expression for the n^{th} term of an infinite sequence, if the first four terms are:

$$\frac{2}{3}, \frac{-4}{5}, \frac{6}{7}, \frac{-8}{9}.$$

3. If $\lim\limits_{n\to\infty} a_n = k_1$, $\lim\limits_{n\to\infty} b_n = k_2 \neq 0$, $\lim\limits_{n\to\infty} c_n = k_3$ determine each of the following limits:

a. $\lim\limits_{n\to\infty} \frac{a_n}{b_n} =$

b. $\lim\limits_{n\to\infty} 2a_n =$

c. $\lim\limits_{n\to\infty} (c_n - a_n) =$

4. Determine the limit if it exists. If no limit, write "none".

a. $\lim\limits_{n\to\infty} \left[2 + \frac{3}{5}\right]^n =$

b. $\lim\limits_{n\to\infty} \frac{3n^2}{2n + 3} =$

c. $\lim\limits_{n\to\infty} \frac{n + 100}{2n^2} =$

5. Write the first four terms of the sequence of partial sums associated with the series:

$$\sum_{i=1}^{\infty} 2^{-i} x^{2i-1}$$

6. Write the first four terms in the expansion of $\left(1 + \frac{2}{x}\right)^{14}$

MATHEMATICS IV

Answers to Suggested Test Questions

MAJOR AREA I

(1.) T; (2.) F; (3.) T; (4.) F; (5.) T; (6.) drawing; (7.) drawing; (8.) drawing;

(9.) $\{-1\}$; (10.) $\{-28\}$; (11.) $\left\{\frac{1+\sqrt{7}}{2}\right\}, \left\{\frac{1-\sqrt{7}}{2}\right\}$; (12.) $\left\{\frac{\pm\sqrt{9-3x}}{2} \text{ for } x \le 3\right\}$.

	Domain	Range	Is it a function?
(13.)	$\{x: x \in \text{Reals}\}$	$\{y : y \le 0\}$	yes
(14.)	$\{x: x \ge 0\}$	$\{y : y \in \text{Reals}\}$	no
(15.)	$\{x: x \in \text{Reals}\}$	$\{y : y \in \text{Reals}\}$	yes

(16.) 11; (17.) $\frac{x^2 + 2x-2}{2x-1}$; (18.) x^2; (19.) $x^3 -3x^2 +4$; (20.) $x^2 -5x+4$

MAJOR AREA II

(1.) 4; (2.) 1; (3.) 4; (4.) 0; (5a.) $\frac{-\sqrt{2}}{2}$; (5b.) 0; (5c.) $\frac{-\sqrt{3}}{2}$;

(6a.) 4π; (6b.) $\frac{11\pi}{6}$; (6c.) $\frac{3\pi}{4}$; (7) b; (8a.) $\frac{21}{191}$; (8b.) $\frac{8}{17}$;

(9.) a,c ; (10a.) $-\frac{3}{4}$; (10b.) $-\frac{12}{5}$; (11.) b,c ; (12.) c ; (13.) $-\frac{24}{25}$.

MAJOR AREA III

(1.) c; (2.) c; (3.) c ; (4.) b ; (5.) e ; (6.) c ; (7.) f ; (8.) $2, 2\pi$;

(9.) $3, \pi$; (10a.) $\frac{\pi}{8}, \frac{-3\pi}{8}$; (10b.) $\frac{s\pi}{6}, -\frac{\pi}{6}$.

MAJOR AREA IV

(1.) $\left\{\frac{3\pi}{4}, \frac{s\pi}{4}\right\}$; (2.) $\left\{\frac{2\pi}{3}, \frac{5\pi}{3}\right\}$; (3.) $\left\{\frac{\pi}{3}, \frac{2\pi}{3}\right\}$; (4.) $\left\{\frac{\pi}{3}, \frac{2\pi}{3}\right\}$;

(5.) $\left\{\frac{\pi}{6}, \frac{7\pi}{6}\right\}$; (6.) $\{0, \pi, 2\pi\}$; (7.) $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}$;

(8.) $\left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$; (9.) $\{1\}$; (10.) C ; (11.) B ; (12.) C ; (13.) B ; (14.) B.

MAJOR AREA V

(1.) 660° ; (2.) 75° ; (3.) $32^\circ 24'$; (4a.) $\{\theta : \theta \text{ is any angle}\}$; (4b.) $\{y : |y| \le 2\}$;

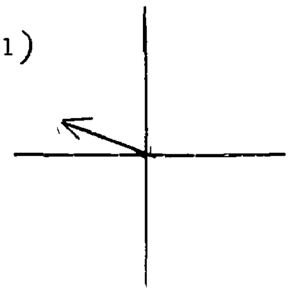
(5a.) $\cos 60^\circ$; (5b.) $\sin 45^\circ$; (5c.) $-\tan 55^\circ$; (6a.) $\frac{4}{5}$; (6b.) $-\frac{3}{5}$;

(6c.) $-\frac{4}{3}$; (7a.) $-\frac{-\sqrt{5}}{5}$; (7b.) $\frac{2\sqrt{5}}{5}$; (7c.) $-\frac{1}{2}$; (8.) $10\sqrt{3}$;

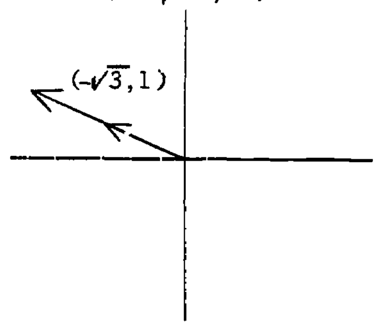
(9.) $\sqrt{380.8}$; (10.) $200\sqrt{37}$ miles.

MAJOR AREA VI

(1a.) 2 ; (1b.) 150° ; (1c.) $(-\sqrt{3}, 1)$



; (1d.) $(-2\sqrt{3}, 2)$



(2a.) $\left(-\frac{5}{2}, -\frac{5\sqrt{3}}{2}\right)$;

(2b.) $(0, \sqrt{2})$; (3a.) $(4,-2)$;

(3b.) $(11, -3)$; (3c.) 0 ; (3d.) $(-9,7)$;

(4a.) 45° ; (4b.) 90° .

MAJOR AREA VII

(1.) c ; (2.) f,i,h,c,k,b,a,k,d ; (3.) $x^2 + y^2 = 25$ or $\pm\sqrt{x^2 + y^2} = 5$;

(4.) $\rho(\cos\theta + \sin\theta) = 4$; (5.) $\left(-\frac{5\sqrt{3}}{2}, \frac{5}{2}\right)$.

MAJOR AREA VIII

(1a.) $(-6,2)$; (1b.) $\frac{3}{20}$; $\frac{1}{20}$; (1c.) $(6,2)$; (2a.) $(-1,4)$; (2b.) $(7,-6)$;

(2c.) $(-7,19)$; (3a.) $-2-i$; (3b.) $\frac{21+i}{34}$ or $\frac{21}{34} + \frac{i}{34}$; (4a.) c or i ;

(4b.) d or $-i$; (4c.) c or i ; (4d.) c or i ; (4e.) d or $-i$; (4f.) a or 1 ;

(4g.) b or -1 ; (4h.) b or -1 ; (5a.) $y = 2$ and $x = 10$; (5b.) $y = 4$ and $x = 3$;

(5c.) $x = \frac{2\pi}{3}$; (6a) $2\sqrt{2}\,(\cos 135^{\circ} + i \sin 135^{\circ})$; (6b.) $2(\cos 240^{\circ} + i \sin 240^{\circ})$.

MAJOR AREA IX

(1.) $-\frac{1}{2}, \frac{-1}{2}, \frac{1}{2}, \frac{1}{2}$; (2.) $\left(\frac{(-1)^{n+1}\,2n}{2n+1}\right)$; (3a.) $\frac{k_1}{k_2}$:

(3b.) $2k_1$; (3c.) $k_3 - k_1$; (4a.) 2 ; (4b.) none ; (4c.) 0 ;

(5.) $\frac{x}{2}, \frac{2x+x^3}{4}, \frac{4x+2x^3+x^5}{8}, \frac{8x+4x^3+2x^5+x^7}{16}$;

(6.) $1+28x^{-1} + 14\cdot 26\, x^{-2} + 16\cdot 14\cdot 13\, x^{-3}$.

MATHEMATICS IV

B-I-B-L-I-O-G-R-A-P-H-Y

1. Butler, Charles H. and F.L. Wren. Trigonometry for Secondary Schools. 3rd ed. Boston, Massachusetts: D.C. Heath Company, 1963.

2. Camp, E.J. Mathematical Analysis. Boston, Mass.: D.C. Heath Company, 1956.

3. Dolciani, Mary, et.al. Modern Introductory Analysis. Boston, Mass.: Houghton-Mifflin Company, 1964.

4. Fuller, Gordon. Analytic Geometry. 2nd Printing. Reading, Massachusetts: Addison-Wesley Company, Inc., 1962.

5. Heineman, E. Richard. Plane Trigonometry with Tables. 3rd ed. New York, New York: McGraw-Hill Book Company, 1964.

6. Meserve, Bruce E., Anthony J. Pettofrezzo, and Dorothy T. Meserve. Principles of Advanced Mathematics. Chicago, Illinois: The L.W. Singer Company, 1964.

7. Murdoch, David C. Analytic Geometry. New York, New York: John Wiley & Sons, Inc., 1966.

8. Nichols, Eugene D. and Henry E. Garland. Modern Trigonometry. New York, New York: Holt, Rinehart and Winston, Inc., 1968.

9. Report of the Commission on Mathematics. Appendices. New York, New York: College Entrance Examination Board, 1959.

10. School Mathematics Study Group. Elementary Functions. New York, New York: Yale University, 1961.

11. Spitzbart, Abraham and Ross H. Bardell. Plane Trigonometry. Reading, Massachusetts: Addison-Wesley Publishing Company, Inc., 1964.

12. Welchons, A.M., W.R. Krickenberger and Helen R. Pearson. Modern Trigonometry. Chicago, Illinois: Ginn and Company, 1962.

13. Wooton, William, Edwin F. Beckenbach, and Mary P. Dolciani. Modern Trigonometry. Boston, Massachusetts: Houghton-Mifflin Company, 1966.